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ANOMALOUS MICROWAVE PROPAGATION ASSESSMENT IN THE LOWER TROPOSP--ETC(U)
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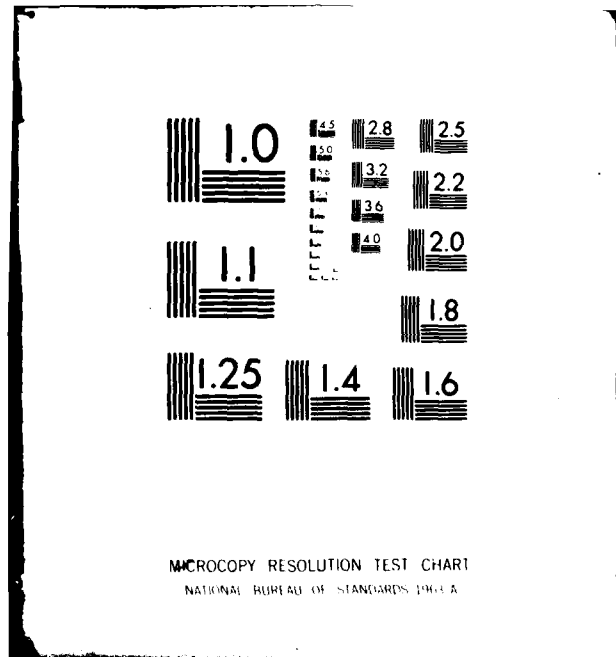
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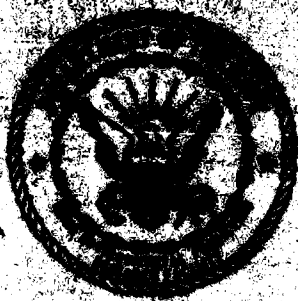
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LEVEL II

**ANOMALOUS MICROWAVE PROPAGATION AS
IN THE LOWER TROPOSPHERE USING
BULK METEOROLOGICAL PARAMETE**

NAVENVPRDRESCHFAC TR 00-01

ADA083156

Wayne Sweet

Naval Environmental Prediction Research Facility
Monterey, California 93840

FEBRUARY 1980

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In their 1967 study "On the Influence of the Meteorological Conditions on the Radiation Properties of Long Range Radars and on the Field Strength From a Distant Radio Transmitter," Gjessing and Moene (G&M) developed a procedure which uses a ΔN parameter evaluated at the 850 mb level to assess the propagation properties for microwave transmissions. This present study examines the skill		

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20. Abstract (continued)

of the G&M procedure to determine resulting accuracy, optimum value of ΔN , and preferred vertical region of assessment.

EASTPAC radiosonde soundings are used to determine the refractive structure in the given vertical region, and to calculate the value of ΔN (850 mb). The G&M procedure's skill is found to be comparable to, or in some cases better than, the skill of present day weather forecasts. The G&M procedure is found to have skill in assessing the existence of anomalous propagation between the surface and the 850 mb level with no indication to the vertical location within this region. The optimum value of ΔN is found to be similar to that used by Gjessing and Moene.

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1. INTRODUCTION

The Navy has an urgent need to determine microwave propagation conditions over data-sparse ocean areas for both real-time and forecast periods. Any radiosonde measurements available to Fleet units are usually taken only from carriers, however, and then on a noninterfering basis. Thus regular assessment and forecasting of anomalous propagation over the ocean areas must rely on large scale numerical model analysis and forecast fields.

The procedure discussed in this report -- developed by Gjessing and Moene (1967)* and using 850 mb data -- has been shown to be effective in assessing the existence of anomalous propagation in certain regions. It can also be used as a forecasting aid by application to forecast fields (prognoses).

The Gjessing and Moene procedure is used to assess and/or forecast anomalous propagation from the surface up to the 850 mb level; this anomalous propagation is then classified as either ducting, or ducting-or-superrefraction. The vertical location of the refractive layer in the surface-to-850 mb region can not be determined by applying the procedure.

Gjessing and Moene used radiosonde data to calculate the 850 mb parameter, and used received-signal data to determine propagation conditions. They claimed 80% accuracy in assessing and forecasting extended radar ranges in the North Sea and in the Mediterranean Sea. This present report reviews and analyzes their method using radiosonde data gathered off the west coast of the United States.

*See References.

2. WORK OF GJESSING AND MOENE

The Gjessing and Moene procedure for assessing and forecasting anomalous propagation (AP) of surface radars over ocean regions uses signal intensity data as a basis for classifying daily propagation characteristics as being either normal or anomalous. The investigators obtained their signal data by positioning a receiver and recorder at a position beyond the normal radar/radio horizon. The normal scattered signal intensity was found to be about 90 db below 1 mW. The threshold signal level was set at 50 db, and any day in which the signal strength was above this level was then classified as an AP day. This classification of days by signal strength was then compared to a determination based on a parameter which indicated the dryness of the air at the 850 mb level.

The assumption in the use of this parameter was that if dry air resides over a region whose surface layer is normally moist due to a vertical water vapor flux from the ocean surface, then somewhere between these two levels a steep gradient in water vapor probably exists. Such gradients would vary in steepness, providing for superrefractive or trapping conditions. The 850 mb parameter was defined using the wet term of the refractivity, N , equation, as follows:

$$\begin{aligned} N &= N_d + N_w \\ N_d &= 77.6 P/T \\ N_w &= 3.7 \times 10^5 e/T^2 \\ \Delta N &= N_w(T_a) - N_w(T_d) \end{aligned}$$

where

$$\begin{aligned} e(T) &= \text{water vapor pressure (mb);} \\ T_a &= \text{air temperature (°K);} \\ T_d &= \text{dew point temperature (°K); and} \\ P &= \text{pressure (mb)} \end{aligned}$$

Gjessing and Moene found the critical value of ΔN to be 15 N -- if ΔN was larger than 15 N, anomalous propagation was assessed/forecast.

The investigators recorded data for 443 days using L-band radars, and for 391 days using a 1 GHZ radio band. These data were randomly selected from the three years of data for each link and hence are each an independent sample. Both of these transmission data links were over water in the North Sea region. The results of the analysis from these two links are given in the contingency tables 1a and 1b.

Table 1a. L-Band radar.

	Observed Signal		Totals
	AP	Normal	
AP ($\Delta N > 15N$) Calculated	118	16	134
Normal ($\Delta N < 15N$)	24	285	309
Totals	142	301	443

Table 1b. 1 GHZ radio band.

	Observed Signal		Totals
	AP	Normal	
AP ($\Delta N > 15N$) Calculated	33	7	40
Normal ($\Delta N < 15N$)	6	345	351
Totals	39	352	391

As given in Tables 1a and 1b, the total number of days correctly calculated is the sum of the diagonal elements; the percentages correct are 91% and 97% respectively. Of the number of AP days assessed for the 1 GHZ band, 82% were correct (prefigurance*), and 85% of the number of the observed AP days were correctly forecast (post agreement**). Both of these percentages

*Percentage of correct forecasts of the event.

**Percentage of observations forecasted correctly.

are related to operational use of such a procedure since they judge its false alarm rate and accuracy, respectively.

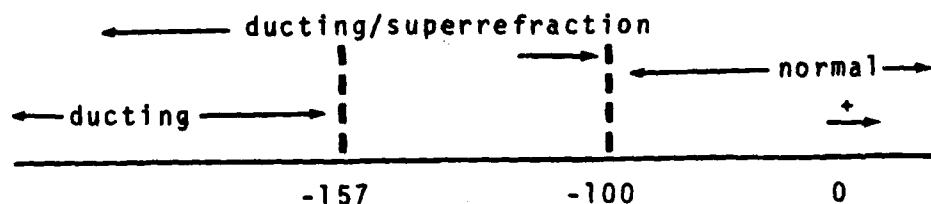
The skill of a forecast procedure also can be judged by comparing its results to the skill of climatology or to chance. The Heidke skill score (Heidke, 1926) is one such method that has been used by the National Weather Service to compare forecast capabilities from year to year (Pierce, 1976). The Heidke skill scores for climatology for the L-band radar and 1 GHZ radio band are .789 and .817, respectively, while the skill scores for normal precipitation forecasts range from .500 to .600 (Pierce, 1976). Compared to these percentages, the skill of the Gjessing and Moene procedure in the North Sea region is evident; other questions remain, however, and these are addressed in this present study.

3. STUDY PROCEDURES

This study was conducted to determine the skill of the ΔN procedure in some region other than the North Sea, to determine independently the critical value of ΔN and to determine the vertical region of greatest skill. Because transmission data such as those used by Gjessing and Moene are difficult to obtain in large quantities, radiosonde data were used for this study. The soundings were analyzed for type of refractive structure in the defined region of assessment. The same soundings were used to calculate the value of ΔN at the 850 mb level.

The calculated value of ΔN was compared to a selected critical value of ΔN , ΔN_c -- if $\Delta N \geq \Delta N_c$, anomalous propagation was assessed. A contingency table was generated and a Heidke skill score was calculated; the data analysis was then iterated on the critical ΔN_c by increments of $2N$, beginning at $8N$. The value of ΔN_c which provided the maximum skill score was taken as the optimum ΔN_c . Figure 1 depicts the assessment region relative to the ΔN determination level.

Several vertical regions of assessment were analyzed to determine if one particular region provided significantly better skill. The layer adjacent to the surface, for example, was examined for the skill of the procedure and also for the effects of the region on the skill of larger regions. Figure 2 shows the seven regions of assessment, where refractive conditions were classified as normal ducting (D) or ducting and/or super-refraction (DSR). The relationships of these three refractive conditions, in terms of N gradients, are diagrammed by below.



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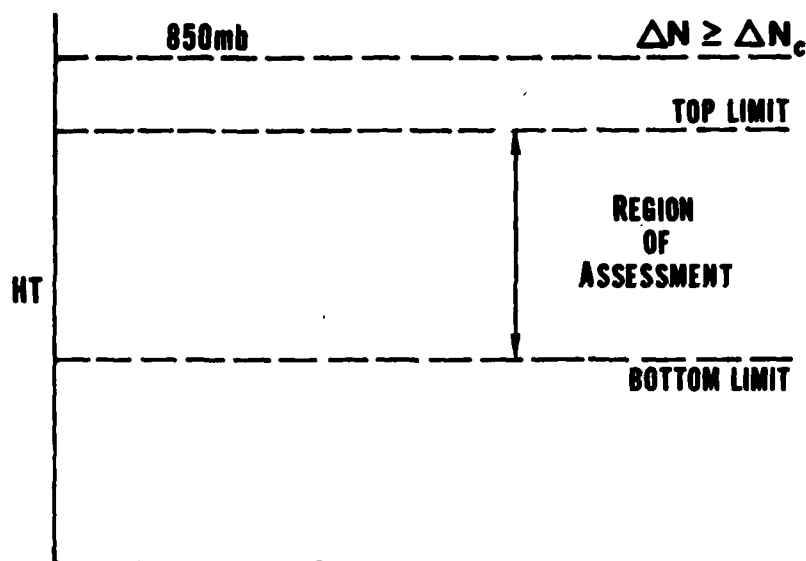


Figure 1. Region of assessment relative to level of ΔN calculation. The analysis procedure iterates on a selected initial ΔN_c . The assessment based on the value of ΔN is related to the anomalous propagation in the region of assessment for all 1525 soundings using 2x2 contingency tables.

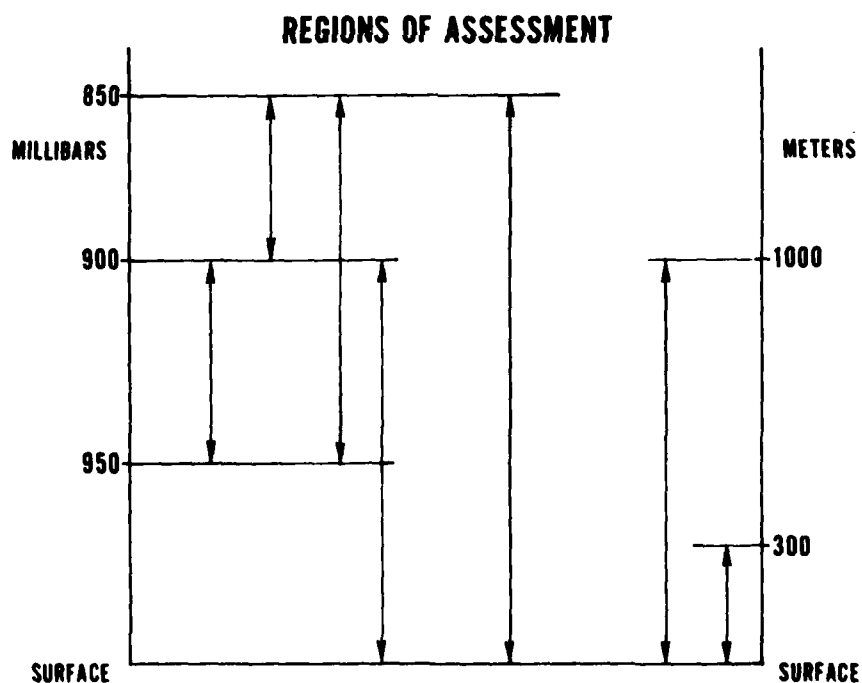


Figure 2. The seven regions of assessment. The primary region, 850 mb to surface, is divided into several sub-regions, two of which are of constant thickness. The constant-pressure difference regions evaluate the procedure's skill in such partial regions.

The radiosonde data used in this study were obtained by radar picket ships stationed off the U.S. west coast during the period 1959-65. More than 6900 soundings were archived by the National Climatic Center; radiosondes were launched from ships stationed between 29°N and 51°N, approximately 200-300 n mi off the coast.

The data were separated into five latitude zones numbered 1-5, with demarcation latitudes at 29°N, 34°N, 38°N, 41°N, 46°N and 51°N (see Figure 3). Data from zone 2, 1500 soundings, were used in this study.

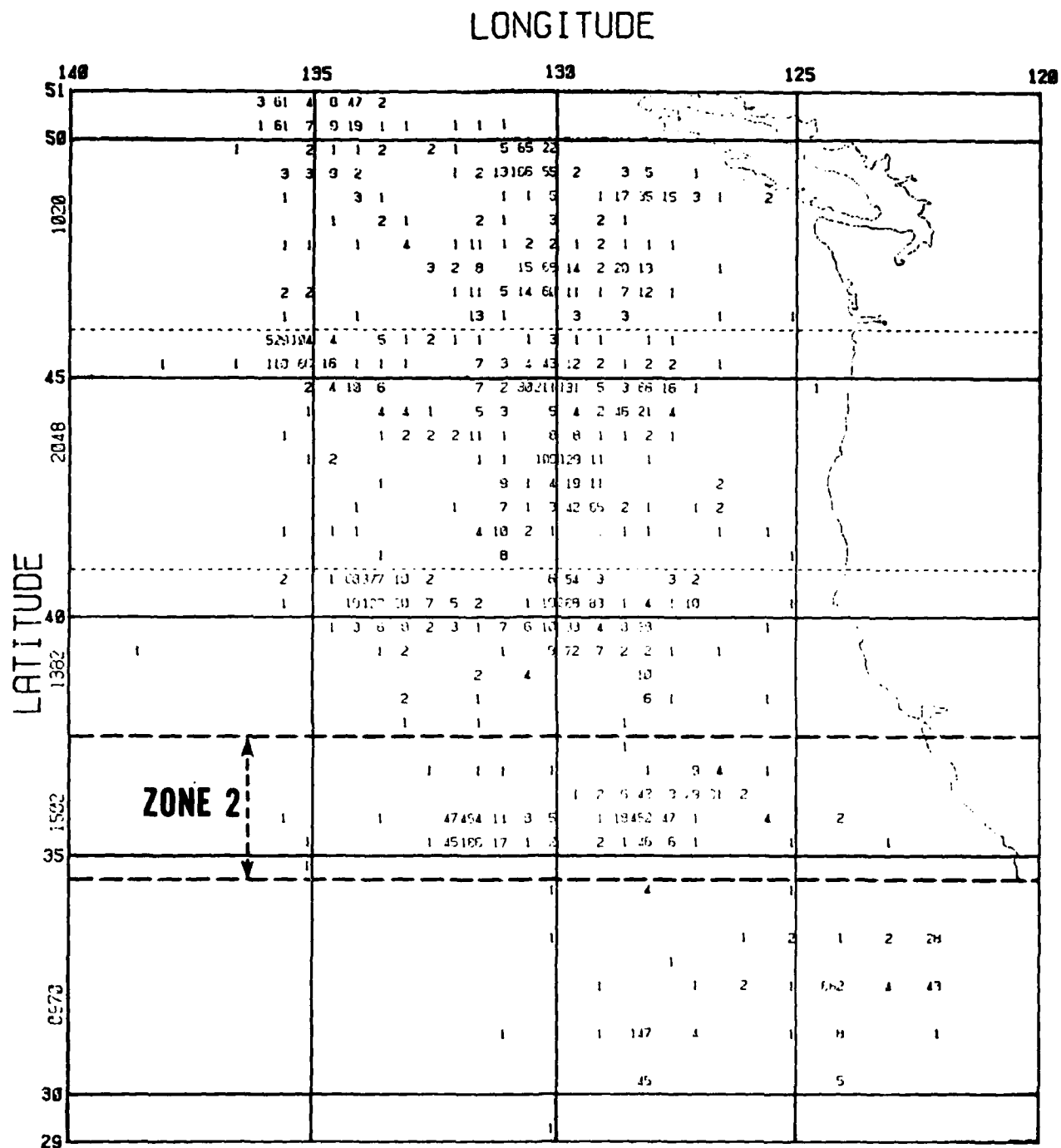


Figure 3. Geographical distribution of radiosonde data, 1959-65.
Data from Zone 2 were used in the present study.

4. RESULTS

4.1 OPTIMUM CRITICAL ΔN

The optimum value of ΔN is defined as the value which corresponds to the highest skill score. To be sure that a true maximum existed for purposes of this study, the iterative procedure was continued well beyond the first maximum. In each case the apparent maximum was the true maximum. Figure 4's two plots show S versus ΔN_c curves for the two types of AP for the 850 mb surface region. The maximum ducting-only occurs in a relatively flat part of the curve and may indeed be hiding a multitude of local maximums.

The existence of a maximum may seem curious at first; the accuracy of assessing AP would seemingly improve with increasing ΔN (an increasingly more restrictive discriminant). The reason for the decrease in skill score after a maximum can be seen from the listings in Table 2, which shows tabulations for the DSR type of AP in the 850-950 mb region.

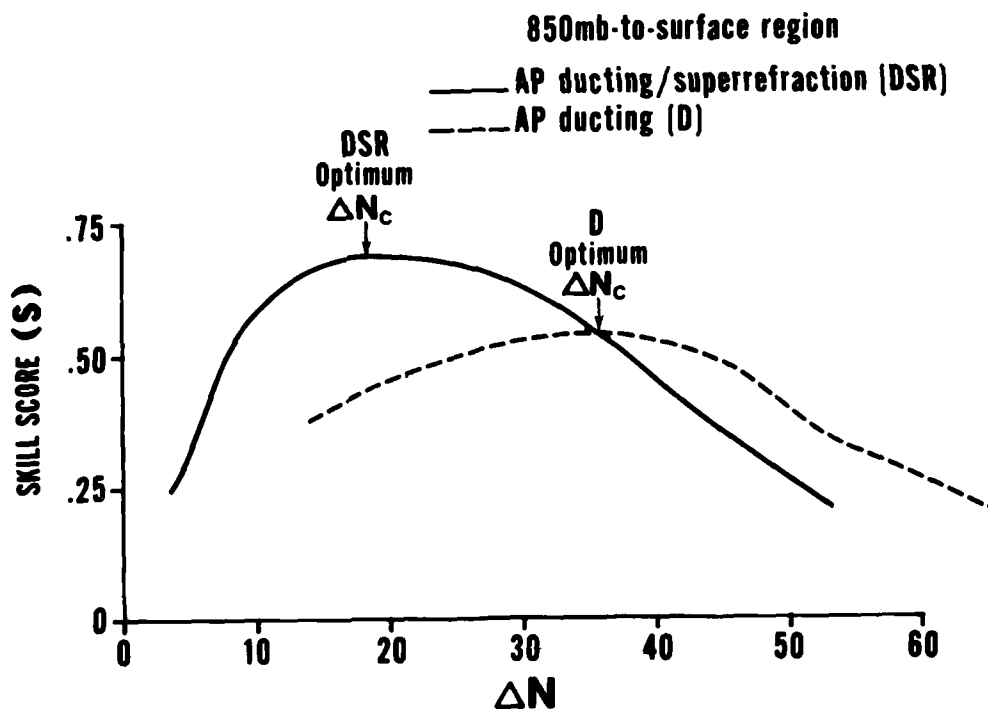


Figure 4. Plot of ΔN_c versus skill score showing that optimum values of ΔN_c exist for ducting/superrefraction AP (solid line) and ducting AP (dashed line).

Table 2. Occurrence of DSR AP in the 850-950 mb region.

ΔN_c	SKILL Score	CORRECT ASSESSMENT (%)		PERCENT Total Correct
		AP	Normal	
10	.568	72	97	79
12	.631	75	97	82
14	.672	78	95	84
16	.703	81	93	85
18	.729	83	93	86
20	.730	84	91	87
22	.727	85	89	86
24	.720	85	87	86
26	.713	87	84	86
28	.694	87	87	85
30	.682	88	80	84

The accuracy of AP assessment steadily increases with increasing ΔN_c , as suspected. Normal assessment accuracy steadily decreases with increasing ΔN_c . The reason for this decreasing accuracy is that more cases of AP are occurring when $\Delta N < \Delta N_c$, because of the higher value of ΔN_c . As ΔN_c exceeds the optimum value, the increase in AP assessment slows, and the loss in accuracy of normal propagation assessment overrides the improving AP assessment. The skill score then begins to decrease, since the score considers both correct and incorrect normal and AP assessments.

4.2 PREFIGURANCE AND POST AGREEMENT

Prefigurance and post agreement percentages indicate false alarm rates and accuracy, respectively, and are defined as (1) the percentage of forecasts of the event that are correct, and (2) the percentage of observations correctly forecasted.

The prefigurance and post-agreement percentages were calculated for all seven regions of assessment and for each of the AP types. The AP prefigurance percentages for the ΔN_c procedure are larger for ducting and/or superrefraction than for ducting alone. The prefigurance percentages are less for the partial regions than for the 850 mb-to-surface region, which indicates a lower false-alarm rate for the 850 mb-to-surface region than for the partial regions. The post agreement percentages are uniformly high, even for those regions having low prefigurance percentages. The tendency, then, is for the ΔN_c procedure to over-forecast the AP event, particularly in the lower-skill regions. These results are given in Table 3.

The assessment of normal propagation conditions have higher prefigurance values than those of AP conditions. The normal prefigurance percentages are higher for the ducting type AP than for the ducting and/or superrefraction type AP. The false-alarm rate therefore is lower for normal conditions than the rate for AP conditions. Table 4 shows the normal prefigurance and post-agreement percentages.

Table 3. Summary of percentages of correct assessments (prefiguration) and correctly assessed observations (post agreement) for anomalous propagation, showing higher percentages for ducting/superrefraction than for ducting.

Assessment Region		AP Type*	AP Prefiguration Percentage	AP Post Agreement Percentage
Top	Bottom			
850 mb	Surface	DSR D	90 67	85 75
850 mb	950 mb	DSR D	84 57	92 83
850 mb	900 mb	DSR D	53 24	93 87
900 mb	Surface	DSR D	67 46	70 74
900 mb	950 mb	DSR D	54 36	85 87
1000 m	Surface	DSR D	58 43	67 66
300 m	Surface	DSR D	21 6	80 71

*D - Ducting
DSR - Ducting and/or superrefraction

Table 4. Summary of percentages of correct assessments (prefigurance) and correctly assessed normal propagation observations (post agreement), showing higher percentages for ducting than for ducting/superrefraction.

Assessment Region		AP Type*	Normal Prefigurance Percentage	Normal Post Agreement Percentage
Top	Bottom			
850 mb	Surface	DSR	76	84
		D	86	79
850 mb	950 mb	DSR	91	80
		D	92	76
850 mb	900 mb	DSR	95	57
		D	97	58
900 mb	Surface	DSR	75	73
		D	90	71
900 mb	950 mb	DSR	93	71
		D	97	69
1000 m	Surface	DSR	77	69
		D	89	75
300 m	Surface	DSR	86	29
		D	95	35

*D - Ducting

DSR - Ducting and/or superrefraction

4.3 SKILL SCORES AND OPTIMUM ΔN_c

The skill scores for the ducting/superrefraction (DSR) type of AP are highest when the upper limit of the assessment region is the 850 mb level. The two highest skill scores are

<u>Region</u>	<u>Skill Score (S)</u>
850 to surface	.67
850 to 950	.73

The increase in S for the region excluding 950 mb to the surface is undoubtedly due to the removal of the area of inaccurate radiosonde data due to ship effects*, and to the lack of skill of the procedure in the lowest region. The region of 300 mb to the surface shows no skill over climatology. The intermediate regions, as tabulated below, have almost identical skill scores, indicating no particular region in which the procedure performs best.

<u>Region</u>	<u>Skill Score (S)</u>
850 to 900	.43
900 to 950	.48
900 to surface	.42

Figure 5 depicts skill scores and optimum ΔN_c for both DSR and ducting AP.

The skill scores of the procedure for ducting are lower than those for DSR. The tabulation below shows the partial regions (the last three) having low scores, with no region preferred.

<u>Region</u>	<u>Skill Score (S)</u>
850 to surface	.53
850 to 950	.52
850 to 900	.22
900 to 950	.36
900 to surface	.38

*Biases in temperature and relative humidity in near-surface data of radiosondes can cause inaccurate profiles of these parameters.

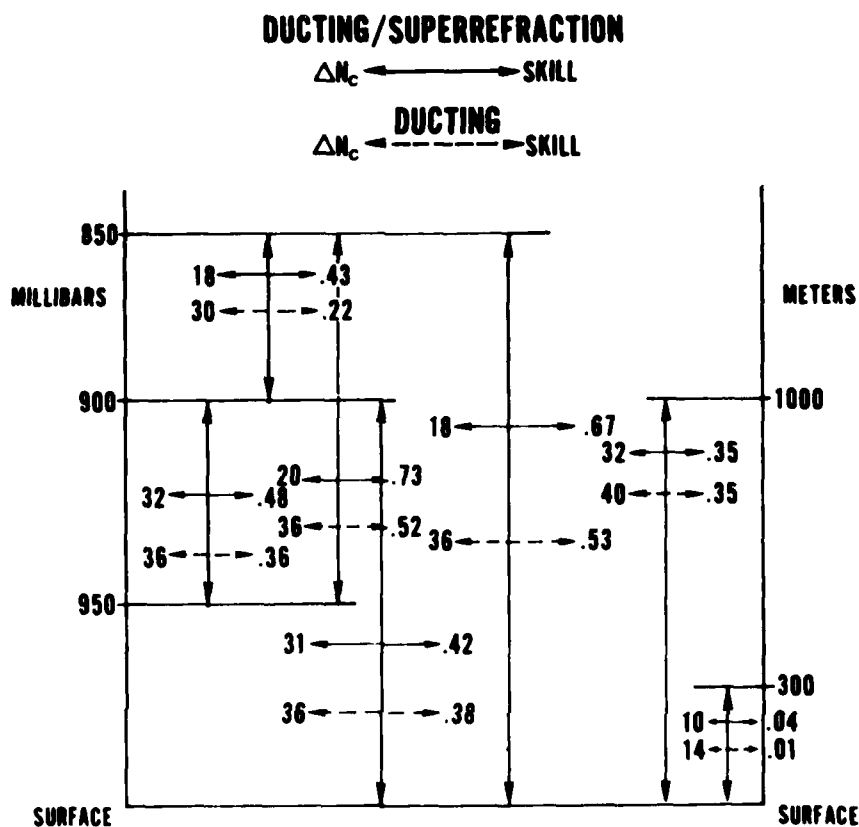


Figure 5. Combined summary for both classes of anomalous propagation, ducting/superrefraction and ducting, showing the larger optimum ΔN_c values for ducting AP. ΔN_c and skill scores for both types of AP are shown for each of the seven assessment regions.

The first two regions, which show the best skill, encompass essentially the same region as that examined by Gjessing and Moene. The tabulated skill scores are considerably below the scores reported by Gjessing and Moene (.79 and .82) because the former are for ducting, which is a more restrictive type of anomalous propagation.

The values of optimum ΔN_c for assessment of ducting are much larger than those for assessment of DSR. This seems reasonable since ducting requires steeper water vapor gradients than does superrefraction. Steeper water vapor gradients would require drier air aloft, thereby making larger ΔN_c values necessary to provide the greatest skill scores. Figure 5 shows that ΔN_c values for ducting are 1 1/4 to 2 times larger than ΔN_c values for DSR.

4.4 COMPARING ΔN PROCEDURE TO SIMPLE CLIMATOLOGY

A simple climatological procedure can be defined as one in which anomalous propagation is always assessed if the climatological probability is greater than 0.5, and normal propagation is always assessed if the climatological probability is less than 0.5. Table 5, which compares the ΔN procedure with simple climatology, indicates that in all cases the ΔN procedure yields better assessments than does simple climatology in ocean areas.

Table 5. Comparison of forecasts by the ΔN_c procedure with forecasts by simple climatology for AP and normal conditions.*

	AP				NORMAL			
	Percent of Forecast Correct		Percent of Occurrences Forecasted		Percent of Forecasts Correct		Percent of Occurrences Forecasted	
$\Delta N \geq 36$ Ducting Surf. Cut-off Climo. = .37	ΔN	Climo	ΔN	Climo	ΔN	Climo	ΔN	Climo
	67	0	75	0	86	63	79	63
$\Delta N \geq 31$ Ducting & SR Surf. Cut-off Climo. = .67	90	67	85	67	76	0	84	0
$\Delta N \geq 36$ Ducting 950 mb cut-off Climo. = .28	57	0	83	0	92	72	76	72
Ducting * SR 950 mb cut-off Climo = .54 $\Delta N \geq 20$	84	54	92	54	41	0	80	0

*Climatological probability >0.5 : AP assessed.
Climatological probability <0.5 : normal propagation assessed.

5. SUMMARY

The assessment of AP by the ΔN procedure achieves its highest Heidke skill for combined ducting and superrefraction from the surface to the 850 mb level. The procedure has very little skill over chance or climatology for the surface-to-300 mb region. The assessment seems to be conservative; that is, the procedure tends to overforecast AP and thus produce a high false-alarm rate. The assessment of normal propagation conditions has a much lower false-alarm rate.

The value of ΔN_c used for discriminating between AP and normal conditions is the value which maximizes the skill score. It was found that a unique maximum existed for each of the assessment/forecast regions. This optimum value of ΔN_c varied from 18 for ducting and superrefraction (DSR) to 36 to 40 for ducting only (D), depending on the region of assessment/forecast. The skill scores increased for both the 850 mb and 900 mb top-cut-off for DSR when the bottom cut-off was set at 950 mb, rather than the surface. The opposite was true for the case of ducting only.

This procedure has no prediction/assessment capability for altitude of the AP structure; there are indications, however, that improved skill scores result when the bottom of the assessment region is elevated off the surface.

6. FUTURE INVESTIGATIONS

The ΔN procedure will be examined using the other four zones of the available picket ship data. Independent data sets will be used to determine the stability of the critical ΔN value. Variations in skill and critical ΔN for each of the five zones will be examined. The procedure will be converted from the categorical (yes-no) assessment to probability assessment. Use of the procedure will be evaluated in other ocean areas where radio-sonde data are available.

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INFORMATION



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22 May 1981

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To: Distribution

Subj: NAVENVPREDRSCHFAC Technical Reports; changes in

1. Subject reports in which pen and ink changes should be made are:

- a. TR 79-01, June 1979: Monthly climatology for evaporation duct occurrence in the North Atlantic Ocean
- b. TR 79-02, July 1979: Summary of an EASTPAC refractive structure climatology
- AD-A083156-c. TR 80-01, February 1980: Anomalous microwave propagation assessment in the lower troposphere using a bulk meteorological parameter
- d. TR 80-02, July 1980: Meteorological factors affecting evaporation duct height climatologies
- e. TR 80-05, October 1980: Assessment/forecasting of anomalous microwave propagation in the troposphere using model output

2. On DD Forms 1473 of all subject reports listed in Para. 1 above,

Block 10 should read . . . PE62759N

Block 11 should read . . . Naval Ocean Systems Center
San Diego, CA 92152

Block 14 should read . . . Naval Material Command
Department of the Navy
Washington, DC 20360

3. On p. 5 of TR 80-05,

Eq. (1) should read $\Delta N = N_w(T_a) - N_w(T_d)$

Eq. (2) should read $\Delta N = B \left[\frac{e(T_a)}{T_a^2} - \frac{e(T_d)}{T_d^2} \right] + \frac{B \Delta e}{T_a^2}$

adding Δ in Eq. (1), and deleting repeated expression $-\frac{e(T_a)}{T_a^2}$ in Eq. (2).

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